Spontaneous Symmetry Breaking in the Space-time of an Arbitrary Dimension

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Abstract

We propose a new scenario to implement spontaneous symmetry breaking in the space-time of an arbitrary dimension ($D > 2$) by introducing the non-minimal coupling between the scalar field and the gravity. In this scenario, the usage of the familiar $\lambda\Phi^4$ term, which is non-renormalizable for $D \geq 5$, can be avoided altogether.

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1 Introduction

In physics, the standard way to achieve *spontaneous symmetry breaking* (SSB) is by introducing a scalar field[1] with a potential like

$$V(\Phi) = -\frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4. \tag{1}$$

Both the negative (mass)^2 Φ^2 term and $\lambda\Phi^4$ term play a crucial role to make the potential behaving like Figure 1 or Figure 2 and possessing multiple true vacua. In the process of choosing one among these true vacua, SSB is achieved and, in the meanwhile, some topological defects like domain walls or cosmic strings[2] could form.

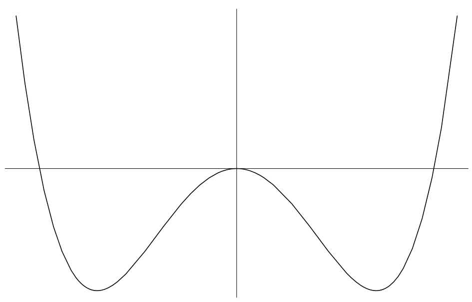

Figure 1: Potential of a single real scalar field responsible for SSB

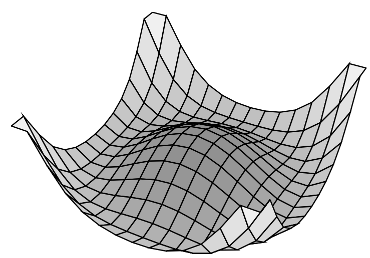

Figure 2: Potential of a complex scalar field responsible for SSB

The *non-minimal coupling* between scalar fields and gravity has been introduced in various topics in quantum field theories and cosmology. In quantum field theories (for a comprehensive review and a list of references, see the book [2]), the conformal invariance of the massless scalar field with the non-minimal coupling constant $\xi = 1/6$ was first noted by Penrose [3] (see also [4]). In the framework of quantum field theories in curved space-time, the non-minimal coupling term can also be introduced by the requirement of renormalizability [5] (for a review, see [6]). In cosmology (see [7] [8] and references therein), the usage of the non-minimal coupling in the inflation

[1] A famous example is the Higgs mechanism [1] in the Standard Model to break the SU(2) $\times$U(1) gauge symmetry spontaneously.

[2] If Φ is a single real scalar field, domain walls can form along with SSB. On the other hand, if Φ denotes a complex scalar field or N (≥ 2) real scalar fields, (cosmic) strings can form.

models [9] and models for the cosmological constant problem [7] [10] (first proposed by Dolgov [11]) has been widely explored. In addition, the non-minimal coupling is also involved in an interesting model called "induced gravity" which was first proposed by Zee in 1979 [12] and is still explored nowadays (e.g. a series of work about induced gravity and inflation by Kao [13]).

The role of the *non-minimal coupling* in SSB and phase transitions in 4D space-time has been widely discussed (for a review, see [2]). It has been pointed out that the non-minimal coupling with the 'external' gravitational field may lead to SSB [14]. It has also been noted that SSB and phase transitions can be induced by curvature via the non-minimal coupling with the 'external' gravitational field [15]. Accompanying the quest of symmetry breaking and vacuum stability in curved space-time [16], that the curvature can be a symmetry-breaking factor has been shown. Note that, in the papers mentioned above, the effect of the curved space-time is taken into account by introducing an 'external' gravitational field, which means that the metric tensor of the space-time is treated as a background, and the *Ricci scalar* $\mathcal{R}$ in the non-minimal coupling term is regarded as an 'external' parameter (like the temperature of a macroscopic system in thermodynamics). This is different from our usage of the gravitational field to be discussed below.

In this paper, we propose an alternative way to achieve SSB in 4D and arbitrary higher ($D \geq 5$) dimensional space-time. We will show that, by introducing the *non-minimal coupling* between the scalar field and gravity and keeping the negative $(\text{mass})^2\ \Phi^2$ term, we can build up a new scenario to implement SSB without using $\lambda\Phi^4$ term. Avoiding the usage of $\lambda\Phi^4$ term could be a benefit when we consider SSB in the higher dimensional space-time, because $\lambda\Phi^4$ term is non-renormalizable in the formalism of quantum field theories in $D \geq 5$ dimensional space-time. The gravitational field introduced in this paper is not an 'external' field or a background field, but strongly depends on the scalar field through the Einstein equations. Such a strong dependence plays a crucial role in implementing SSB in our scenario. This is one of the essential differences between the previous work mentioned in last paragraph and our work in this paper.

To illustrate our idea specifically, we consider a simple case in the third part of this paper: a world with an arbitrary space-time dimension which is dominated by the 'cosmological constant', the 'vacuum energy', and the

'potential energy'[3]. (In fact, the space-time dimension D should be larger than two in our discussion.) In this simple case, we will see that the potential of the scalar field entails multiple true vacua, which is the key element to implement SSB.

2 Non-minimal coupling to gravity

To introduce the non-minimal coupling between the scalar field and gravity, we consider the Lagrangian density of the scalar field Φ in D dimensional space-time,

$$\mathcal{L} = \frac{1}{2}\mathcal{G}^{MN}\left(\partial_M \Phi\right)\left(\partial_N \Phi\right) - V_{\mathcal{R}}\left(\Phi\right), \qquad M, N = 0, 1, ..., (D-1) \tag{2}$$

where

$$V_{\mathcal{R}}\left(\Phi\right) = \frac{1}{2}\xi\mathcal{R}\Phi^2 - \frac{1}{2}\mu^2\Phi^2 \tag{3}$$

is the potential term for the scalar field Φ which includes the non-minimal coupling to gravity via the *Ricci scalar* $\mathcal{R}$ with a coupling constant ξ which is *positive*[4] in our consideration, and $\mathcal{G}^{MN}$ is the metric tensor of the D dimensional space-time.

To discuss the behavior of the potential $V_{\mathcal{R}}(\Phi)$, we need to know the behavior of the *Ricci scalar* $\mathcal{R}$, which can be obtained from the D dimensional

[3] These three terms—cosmological constant, vacuum energy, and potential energy—can be summed up to form a single term which is treated as an effective cosmological constant (or effective vacuum energy) [17] [18]: The energy-momentum tensor of a vacuum in the quantum framework has the same form with that of a cosmological constant, and hence makes a contribution to the effective cosmological constant. In addition, the potential part in the action of the scalar field Φ in the classical framework also contributes a energy-momentum tensor which has the same cosmological-constant form when Φ is constant in space-time.

[4] Note that the convention for the metric tensor in this paper is:

$$\eta_{MN} = (1, -1, -1, ..., -1),$$

which is different from that in some of the papers mentioned in '**Introduction**' so that the sign of the non-minimal coupling constant ξ in this paper is opposite to that in them.

Einstein equations:

$$\begin{aligned} G_{MN} &= \Lambda \mathcal{G}_{MN} + \kappa T_{MN}, \qquad M, N = 0, 1, ..., (D-1) \\ &= \kappa \left(\frac{1}{\kappa} \Lambda \mathcal{G}_{MN} + T_{MN} \right) \\ &\equiv \kappa \tilde{T}_{MN}, \end{aligned} \tag{4}$$

where G_{MN} is the D dimensional Einstein tensor, Λ is the cosmological constant, T_{MN} is the energy-momentum tensor, $\tilde{T}_{MN}$ is the effective energy-momentum tensor[5], and κ is a parameter (to be called "D dimensional gravitational constant") which is positive and has the same role with the Newton's constant G in four dimensional space-time. For simplicity, we consider the case for the perfect fluid with the effective energy-momentum tensor

$$\tilde{T}^M{}_N = diag\left(\tilde{\rho}, -\tilde{p}_1, -\tilde{p}_2, ..., -\tilde{p}_{(D-1)}\right). \tag{5}$$

By taking trace of the Einstein equations, it is straightforward to get a relation between the *Ricci scalar* $\mathcal{R}$ and $\rho, p_1, p_2, ..., p_{(D-1)}$:

$$-\frac{D-2}{2}\mathcal{R} = \kappa Tr\left(\tilde{T}^M{}_N\right) = \kappa\left(\tilde{\rho} - \tilde{p}_1 - \tilde{p}_2 - ... - \tilde{p}_{(D-1)}\right). \tag{6}$$

Consider a simple case that the effective energy-momentum tensor is dominated by the effective cosmological constant Λ_{eff} which includes the original cosmological constant Λ and the possible contribution from the vacuum energy or the potential energy (see footnote 3):

$$\tilde{T}_{MN} \simeq \frac{1}{\kappa}\Lambda_{eff}\mathcal{G}_{MN}, \tag{7}$$

and hence

$$\tilde{\rho} = \frac{1}{\kappa}\Lambda_{eff} = -\tilde{p}_1 = -\tilde{p}_2 = ... = -\tilde{p}_{(D-1)}. \tag{8}$$

Consequently, in this simple case,

$$\mathcal{R} = -\frac{2D}{D-2}\Lambda_{eff}, \tag{9}$$

[5] The cosmological constant term $\Lambda\mathcal{G}_{MN}$ can be absorbed into T_{MN} (as shown in Eq. (4)) to form an effective energy-momentum tensor $\tilde{T}_{MN}$ in which the contribution from $\Lambda\mathcal{G}_{MN}$ is included.

which is positive(negative) for a negative(positive) effective cosmological constant Λ_{eff} and proportional to Λ_{eff}.

The above result is essential in the following discussion about implementing SSB in the third part of this paper. We will see that, if we treat the *Ricci scalar* $\mathcal{R}$ as a parameter and write the potential as

$$V_{\mathcal{R}}(\Phi) = \frac{1}{2}\left(\xi\mathcal{R} - \mu^2\right)\Phi^2, \tag{10}$$

the "coefficient" $(\xi\mathcal{R} - \mu^2)$ can be negative for small Φ^2 and positive for large Φ^2 in some situation. And hence the potential $V_{\mathcal{R}}(\Phi)$ behaves similarly to the well-known potential (Eq. (1)) for Higgs mechanism, and entails multiple true vacua.

To see it more specifically, let us consider a simple case.

3 A simple case to illustrate the way of implementing spontaneous symmetry breaking

We consider the action in which only gravity and the scalar field are introduced,

$$S = -\frac{1}{2\kappa}\int d^D x\sqrt{\mathcal{G}}\left(\mathcal{R} + 2\Lambda\right) + \int d^D x\sqrt{\mathcal{G}}\mathcal{L}, \tag{11}$$

where

$$\mathcal{L} = \frac{1}{2}\mathcal{G}^{MN}\left(\partial_M\Phi\right)\left(\partial_N\Phi\right) - V_{\mathcal{R}}(\Phi), \qquad M, N = 0, 1, ..., (D-1), \tag{12}$$

$$V_{\mathcal{R}}(\Phi) = \frac{1}{2}\xi\mathcal{R}\Phi^2 - \frac{1}{2}\mu^2\Phi^2, \tag{13}$$

κ is the (positive) D dimensional gravitational constant, $\mathcal{G}$ is the absolute value of the determinant of the metric tensor $\mathcal{G}_{MN}$, Λ is the cosmological constant, and ξ is a *positive* coupling constant.

The variation of the above action with respect to the scalar field Φ and the metric tensor $\mathcal{G}^{MN}$ yields the field equation of Φ :

$$\Phi^{;J}{}_{;J} + \left(\xi\mathcal{R} - \mu^2\right)\Phi = 0, \tag{14}$$

and the Einstein equations:

$$\begin{aligned} G_{MN} &= \Lambda \mathcal{G}_{MN} + \kappa \Big\{ (\partial_M \Phi)(\partial_N \Phi) - \mathcal{L}_\Phi^{(0)} \mathcal{G}_{MN} \\ &\qquad - \xi \Phi^2 G_{MN} + \xi \left(\Phi^2\right)_{;M;N} - \xi \left(\Phi^2\right)^{;J}{}_{;J} \mathcal{G}_{MN} \Big\}, \end{aligned} \tag{15}$$

where

$$\mathcal{L}_\Phi^{(0)} = \frac{1}{2} \mathcal{G}^{M'N'} (\partial_{M'} \Phi)(\partial_{N'} \Phi) + \frac{1}{2} \mu^2 \Phi^2. \tag{16}$$

(Note that there is no non-minimal coupling term in $\mathcal{L}_\Phi^{(0)}$.) The semicolon ';' denotes the 'covariant derivative'. In the brace in Eq. (15), the term $-\xi \Phi^2 G_{MN}$, as well as the terms $\xi \left(\Phi^2\right)_{;M;N}$ and $-\xi \left(\Phi^2\right)^{;J}{}_{;J} \mathcal{G}_{MN}$, is derived from the variation of the term $-\frac{1}{2} \xi \mathcal{R} \Phi^2 \sqrt{\mathcal{G}}$ in the action. This term modifies the Einstein equations, the gravitational constant κ and the cosmological constant Λ as follows:

$$\begin{aligned} G_{MN} &= \Lambda' \mathcal{G}_{MN} + \kappa_{eff} \Big\{ (\partial_M \Phi)(\partial_N \Phi) - \mathcal{L}_\Phi^{(0)} \mathcal{G}_{MN} \\ &\qquad + \xi \left(\Phi^2\right)_{;M;N} - \xi \left(\Phi^2\right)^{;J}{}_{;J} \mathcal{G}_{MN} \Big\}, \end{aligned} \tag{17}$$

where the 'modified cosmological constant' Λ' and the 'effective gravitational constant' κ_{eff} are defined as

$$\Lambda' \equiv \frac{\Lambda}{1 + \kappa \xi \Phi^2}, \tag{18}$$

$$\kappa_{eff} \equiv \frac{\kappa}{1 + \kappa \xi \Phi^2}. \tag{19}$$

Note that both Λ' and κ_{eff} are not constant parameters, but have the dependence on the square of the scalar field. Taking trace of both sides of the *modified* Einstein equations (17) gives us the *Ricci scalar* $\mathcal{R}$ as a function of the scalar field Φ and its covariant derivatives:

$$\begin{aligned} \mathcal{R} &= \frac{\kappa}{1 + \kappa \xi \Phi^2} \Big\{ -\frac{2D}{D-2} \left(\frac{1}{\kappa} \Lambda - \frac{1}{2} \mu^2 \Phi^2 \right) \\ &\qquad + \Phi^{;J} \Phi_{;J} + 2 \left(\frac{D-1}{D-2} \right) \xi \left(\Phi^2\right)^{;J}{}_{;J} \Big\}. \end{aligned} \tag{20}$$

For exploring the existence of multiple true vacua entailed by the action (Eq. (11)) or the potential $V_\mathcal{R}(\Phi)$ (Eq. (13)), we need to find out the constant

solutions for the scalar field Φ and explore their stability[6]. For constant Φ, the field equation of Φ (14) and the *modified* Einstein equations (17) become

$$\left(\xi\mathcal{R}-\mu^2\right)\Phi=0, \tag{21}$$

$$\begin{aligned} G_{MN} &= \Lambda'\mathcal{G}_{MN}+\kappa_{eff}\left\{-\frac{1}{2}\mu^2\Phi^2\mathcal{G}_{MN}\right\} \\ &= \frac{\kappa}{1+\kappa\xi\Phi^2}\left\{\frac{1}{\kappa}\Lambda\mathcal{G}_{MN}-\frac{1}{2}\mu^2\Phi^2\mathcal{G}_{MN}\right\} \\ &= \kappa_{eff}V_{eff}\mathcal{G}_{MN}=\Lambda_{eff}\mathcal{G}_{MN}, \end{aligned} \tag{22}$$

where the '*effective* potential energy (or vacuum energy)' V_{eff} and the '*effective* cosmological constant' Λ_{eff} are defined as

$$V_{eff}=\frac{1}{\kappa}\Lambda-\frac{1}{2}\mu^2\Phi^2, \tag{23}$$

$$\Lambda_{eff}=\frac{1}{1+\kappa\xi\Phi^2}\left(\Lambda-\frac{1}{2}\kappa\mu^2\Phi^2\right)=\kappa_{eff}V_{eff}. \tag{24}$$

From Eq. (22), we can see that the constant-Φ solution is corresponding to a Λ-dominated world, and hence the situation here becomes as simple as the case mentioned in last section.

Taking trace of both sides of Eq. (22) (or applying the condition $\Phi=$ constant to Eq. (20)) gives us the *Ricci scalar* $\mathcal{R}$ as a function of the scalar field Φ :

$$\mathcal{R}=-\frac{2D}{(D-2)}\frac{\kappa}{(1+\kappa\xi\Phi^2)}\left(\frac{1}{\kappa}\Lambda-\frac{1}{2}\mu^2\Phi^2\right). \tag{25}$$

Using the above equation, Eq. (21) becomes

$$\left[-\xi\frac{2D}{(D-2)}\frac{\kappa}{(1+\kappa\xi\Phi^2)}\left(\frac{1}{\kappa}\Lambda-\frac{1}{2}\mu^2\Phi^2\right)-\mu^2\right]\Phi=0. \tag{26}$$

Then we can obtain the constant solutions for the scalar field Φ :

$$\Phi=\Phi_{(c)}=0,\ \pm v \tag{27}$$

[6]A vacuum state is usually corresponding to a stable constant-field solution.

where v is defined by

$$v^2 = \frac{D}{\kappa\xi}\left(\frac{D-2}{2D} + \frac{\xi\Lambda}{\mu^2}\right) \quad > 0 \quad \text{for} \quad \xi\Lambda > -\left(\frac{D-2}{2D}\right)\mu^2. \tag{28}$$

The corresponding metric tensor $\mathcal{G}^{(c)}_{MN}$ (the solution(s) of the modified Einstein equations corresponding to $\Phi = \Phi_{(c)}$ (Eq. (22))) will be the metric tensor of de Sitter or anti-de Sitter space-time, depending on the '*effective* cosmological constant' Λ_{eff} is positive or negative.

Now we need to explore the stability of these three constant-Φ solutions ($\Phi = \Phi_{(c)}$, $\mathcal{G}_{MN} = \mathcal{G}^{(c)}_{MN}$). Considering the small variations around these solutions:

$$\begin{cases} \Phi = \Phi_{(c)} + \delta\Phi \\ \mathcal{G}_{MN} = \mathcal{G}^{(c)}_{MN} + \delta\mathcal{G}_{MN} \quad \longrightarrow \quad \mathcal{R} = \mathcal{R}_{(c)} + \delta\mathcal{R} \end{cases}, \tag{29}$$

where $\mathcal{R}_{(c)}$ is the *Ricci scalar* for $\Phi = \Phi_{(c)}$, the field equation of Φ (14) becomes (up to $\mathcal{O}(\delta\Phi, \delta\mathcal{R})$):

$$(\delta\Phi)^{;J}{}_{;J} + \left(\xi\mathcal{R}_{(c)} - \mu^2\right)\delta\Phi + \xi\Phi_{(c)}\delta\mathcal{R} = 0, \tag{30}$$

while the equation for the *Ricci scalar* $\mathcal{R}$ (20) becomes (up to $\mathcal{O}(\delta\Phi, \delta\mathcal{R})$):

$$\delta\mathcal{R} = 0 \quad \text{for} \quad \Phi_{(c)} = 0, \quad \text{or} \tag{31}$$

$$\delta\mathcal{R} = 2\left(\frac{D-1}{D-2}\right)\left(\frac{\kappa\mu^2\Phi_{(c)}}{1+\kappa\xi\Phi^2_{(c)}}\right)\delta\Phi + 4\left(\frac{D-1}{D-2}\right)\xi\Phi_{(c)}\,(\delta\Phi)^{;J}{}_{;J} \quad \text{for} \quad \Phi_{(c)} = \pm v. \tag{32}$$

Using above equations and Eq. (25), we can obtain the field equations of $\delta\Phi$ from Eq. (30): For $\Phi_{(c)} = 0$,

$$(\delta\Phi)^{;J}{}_{;J} + \left[-\left(\frac{2D}{D-2}\xi\Lambda + \mu^2\right)\right]\delta\Phi = 0, \tag{33}$$

where

$$-\left(\frac{2D}{D-2}\xi\Lambda + \mu^2\right) \lesseqgtr 0 \quad \text{for} \quad \xi\Lambda \gtrless -\left(\frac{D-2}{2D}\right)\mu^2. \tag{34}$$

So the solution $\Phi = 0$ is unstable or stable for $\xi\Lambda$ is greater or smaller than $-\left(\frac{D-2}{2D}\right)\mu^2$. (Note that $\xi\Lambda > -\left(\frac{D-2}{2D}\right)\mu^2$ is the same condition with the one for $v^2 > 0$). On the other hand, for $\Phi_{(c)} = \pm v$,

$$(\delta\Phi)^{;J}{}_{;J} + 2\left(\frac{D-1}{D-2}\right)\left[1 + 4\left(\frac{D-1}{D-2}\right)\xi^2 v^2\right]^{-1}\left(\frac{\kappa\xi\mu^2 v^2}{1+\kappa\xi v^2}\right)\delta\Phi = 0, \quad (35)$$

where

$$2\left(\frac{D-1}{D-2}\right)\left[1 + 4\left(\frac{D-1}{D-2}\right)\xi^2 v^2\right]^{-1}\left(\frac{\kappa\xi\mu^2 v^2}{1+\kappa\xi v^2}\right) > 0 \quad \text{for} \quad D > 2. \quad (36)$$

So the solutions $\Phi = \pm v$ are stable for $D > 2$. (Remember that, from Eq. (28), the condition $\xi\Lambda > -\left(\frac{D-2}{2D}\right)\mu^2$ should be fulfilled in order to ensure the existence of the solutions $\Phi = \pm v$.)

Consequently we can conclude that, considering the space-time dimension $D > 2$, for $\xi\Lambda < -\left(\frac{D-2}{2D}\right)\mu^2$, there is only one constant solution $\Phi = 0$, which is stable and corresponding to one true vacuum; while for $\xi\Lambda > -\left(\frac{D-2}{2D}\right)\mu^2$, there are one unstable constant solution $\Phi = 0$ and two stable constant solutions $\Phi = \pm v$ which are corresponding to two true vacua.

4 Discussion

We have shown that the potential $V_{\mathcal{R}}(\Phi)$ does entail multiple true vacua and hence be able to result in spontaneous symmetry breaking under suitable conditions. In particular, we can see, from Eq. (28)(34)(36), that the conditions

$$\left\{\begin{array}{lcl} D & > & 2 \\ \kappa & > & 0 \\ \xi & > & 0 \\ \xi\Lambda & > & -\left(\frac{D-2}{2D}\right)\mu^2 \end{array}\right. \quad (37)$$

should be fulfilled in order to produce spontaneous symmetry breaking in our scenario.

In our scenario, the gravitational constant κ will undergo a "phase transition" accompanying the spontaneous symmetry breaking:

$$\kappa \xrightarrow{\text{SSB}} \kappa_{eff} = \frac{\kappa}{1+\kappa\xi v^2}$$

when the scalar field Φ rolls down to one of the true vacua from the origin. It is interesting to see the large $\xi\Lambda$ limit of κ_{eff}:

$$\xi\Lambda \gg \mu^2, \qquad \kappa_{eff} \simeq \left(\frac{\mu^2}{D\xi\Lambda}\right)\kappa \ll \kappa.$$

Accordingly, even though the original gravitational constant κ could be arbitrarily large, we still can get a small effective gravitational constant κ_{eff} by choosing (or tuning) the parameters: μ, ξ, and Λ such that the fraction $\mu^2/\xi\Lambda$ is small enough. This phenomenon is similar to the result of the "induced gravity" model [12].

It would be of interest to find, numerically, the solutions in our model, that is, the solutions of the coupled equations — the field equation of the scalar field Φ (14) and the modified Einstein equations (17)[7]. Except the trivial solutions: $\Phi = \Phi_{(c)}$, $\mathcal{G}_{MN} = \mathcal{G}^{(c)}_{MN}$, which have been discussed in last section, we want to find out non-trivial solutions, especially the domain-wall solution(s) for studying the domain-wall formation under our new SSB scenario and the relationship between such kind of domain wall and the Randall-Sundrum scenario [19] [20] in 5D space-time. In our preliminary sight on it, we do see that the configuration of this domain wall is similar to the configuration of the brane world in the Randall-Sundrum scenario [19], in which the appropriate "Friedmann equation(s)" (describing the expansion or contraction of the brane world) can be obtained successfully [21]. We also wish to explore more details on it.

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[7]In fact, there is one redundant equation among them.

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